5th Year

Algebra

Class 2

Roots of Quadratic equations and Discriminants

Comment on the nature of the roots of the following equations:

① (i) $x^2-x-6=0$ (ii) $x^2+10x+25=0$ (iii) $x^2-4x+5=0$

② If $(b+1)x^2+bx+b+1=0$ has 2 equal real roots, find the values of b.

③ Show that $x^2-3kx+(k^2-6)=0$ has real roots.

④ Prove that the equation $(a-2)x^2+2x-a=0$ has real and distinct roots.

⑤ Prove that the roots of $kx^2+(2k-1)x-2=0$ are real.

⑥ Find the values of a for which $x^2+ax+16=0$ has real roots.

⑦ $(2p+1)x^2+(p+2)x+1=0$ has real roots. Find the range of values of p.

⑧ Show that $4ax^2-4ax+a+c^2=0$ has no real roots, $a\in N$.

⑨ Find the range of values of c if $cx^2+4x-2=0$ has no real roots.

Algebra Homework 5th Year

Class 2

❶ If x^2+0x-t is a factor of x^3-px^2-qx+r, show that $pq=r$.

❷ $f(x)= x^3-(h+2)x +2k$, $p(x)= 2x^3+hx^2-4x-k$. Given that $(x+3)$ is a common factor of $f(x)$ and $g(x)$, Find the value of h and the value of k.

❸ Show that the roots of the equation $x^2-3x+2-c^2=0$ are real.

❹ Show that the roots of $px^2+(p+q)x+q=0$ are real.

❺ x^2+ax+b is a factor of x^3+qx^2+rx+s, prove that $r-b=a(q-a)$ and $s=b(q-a)$.

① x^2+0x-t is a factor of x^3-px^2-qx+r, show $pq=r$

$$\begin{array}{r} x+k \\ x^2+0x-t\,\overline{\smash{)}\,x^3-px^2-qx+r} \end{array}$$

let $(x+k)(x^2+0x-t) = x^3-px^2-qx+r$

$$x^3+0x^2-tx+kx^2+0x-kt = x^3-px^2-qx+r$$

$\underline{\underline{x^2}}$	$\underline{\underline{x}}$	$\underline{\underline{\text{constants}}}$
$0+k=-p$	$-t+0=-q$	$-kt=r$
$\underline{\underline{k}}=-p$	$-t=-q$	
	$\underline{\underline{t}}=q$	

fill these in

$\underline{\text{So}}$

$$-(-p)(q)=r$$

$$pq=r$$

Proven.

② $x+3$ is a common factor of $f(x)$ and $p(x)$

$\therefore$ $x=-3$ is a common root

fill in $x=-3$ to both equations

$$f(x) = x^3 - (h+2)x + 2k$$

$$f(-3) = (-3)^3 - (h+2)(-3) + 2k = 0$$

$$-27 + 3(h+2) + 2k = 0$$

$$-27 + 3h + 6 + 2k = 0$$

$$\boxed{3h + 2k - 21 = 0}$$

$$p(x) = 2x^3 + hx^2 - 4x - k$$

$$p(-3) = 2(-3)^3 + h(-3)^2 - 4(-3) - k = 0$$

$$-54 + 9h + 12 - k = 0$$

$$\boxed{9h - k - 42 = 0}$$

Now Simultaneous Equations

$$3h + 2k - 21 = 0$$

$$9h - k - 42 = 0 \quad (\times 2)$$

$$3h + 2\cancel{k} - 21 = 0$$

$$18h - \cancel{2k} - 84 = 0$$

$$21h - 105 = 0$$

$$21h = 105$$

$$\boxed{h=5} \longrightarrow 3(5) + 2k - 21 = 0$$

$$2k = 6$$

$$\boxed{k=3}$$

③ Show the roots of $x^2-3x+2-c^2=0$ are REAL

$$\boxed{\begin{array}{l} a=1 \\ b=-3 \\ c=2-c^2 \end{array}}$$

Apply

$$b^2-4ac \geq 0$$

$$(-3)^2-4(1)(2-c^2) \geq 0$$

$$9-8+4c^2 \geq 0$$

Prove

$$\left. \underset{\underset{\text{pos}}{\uparrow}}{1} + \underset{\underset{\text{pos}}{\uparrow}}{4}\,\underset{\underset{\text{anything squared is positive}}{\uparrow}}{c}{}^2 \geq 0 \right] \text{Try to prove this is positive}$$

Proven it is always positive

$\therefore$ it has real roots.

④ Show that the roots of

$px^2 + (p+q)x + q = 0$ are real

ie. Prove $b^2 - 4ac > 0$

$a = p$

$b = p+q$

$c = q$

$$(p+q)^2 - 4(p)(q) > 0$$

$$(p+q)(p+q) - 4pq > 0$$

$$p^2 + pq + pq + q^2 - 4pq > 0$$

$$p^2 - 2pq + q^2 > 0$$

$$(p - q)^2 > 0$$

Proven since anything squared is positive.

⑤

$$\begin{array}{r} x+k \xleftarrow{\quad} \\ x^2+ax+b\ \overline{)\ x^3+qx^2+rx+s} \end{array}$$

let x+k be ~~the~~ factor on top.

$$(x+k)(x^2+ax+b) = x^3+qx^2+rx+s$$

$$x^3+ax^2+bx+kx^2+akx+bk = x^3+qx^2+rx+s$$

<u>now equal like things</u> ......

$$\begin{tikzcd}
\underline{x^2} & \underline{x} & \underline{\text{constants}} \\
a+k=q & b+ak=r & bk=s \\
\therefore k=q-a \arrow[ru, bend right, "\text{Fill in}"'] \arrow[rru, bend right] & &
\end{tikzcd}$$

$\underline{x}$	$\underline{\text{constants}}$
$b+a(q-a)=r$	$b(q-a)=s$
$\therefore a(q-a)=r-b$	PROVEN
PROVEN	