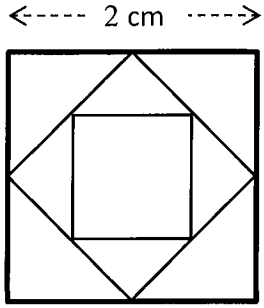
Class 2

Question 4 **(25 marks)**

(a) The amount of a substance remaining in a solution reduces exponentially over time. An experiment measures the percentage of the substance remaining in the solution. The percentage is measured at the same time each day. The data collected over the first 4 days are given in the table below. Based on the data in the table, estimate which is the first day on which the percentage of the substance in the solution will be less than 0·01%.

Day	**1**	**2**	**3**	**4**
Percentage of substance (%)	95	42·75	19·2375	8·6569

(b) A square has sides of length 2 cm. The midpoints of the sides of this square are joined to form another square. This process is continued.
The first three squares in the process are shown below.
Find the sum of the perimeters of the squares if this process is continued indefinitely.
Give your answer in the form $a + b\sqrt{c}$ cm, where a, b, and $c \in \mathbb{N}$.

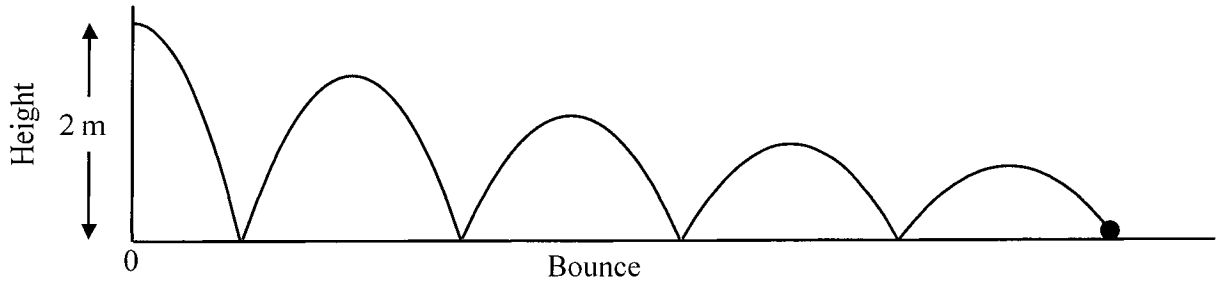


Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

Mary threw a ball onto level ground from a height of 2 m. Each time the ball hit the ground it bounced back up to $\frac{3}{4}$ of the height of the previous bounce, as shown.



(a) Complete the table below to show the maximum height, **in fraction form**, reached by the ball on each of the first four bounces.

Bounce	0	1	2	3	4
Height (m)	$\frac{2}{1}$				

(b) Find, in metres, the total vertical distance (up and down) the ball had travelled when it hit the ground for the 5th time. Give your answer in fraction form.

(c) If the ball were to continue to bounce indefinitely, find, in metres, the total vertical distance it would travel.

Sequences and Series Homework ②

1. If S_n of an arithmetic series is $S_n=3n^2-4n$, find T_7 and T_{15}.

2. The sum of three numbers in geometric sequence is 19 and their product is 216. Find the three numbers.

3. Find a and b if $\frac{1}{4}$, a, 9 are in geometric sequence and $\frac{1}{4}$, a, 9-b are in arithmetic sequence.

4. 4x-4, x+4 , and 2x-7 are three consecutive terms in a sequence where $x \in Z$. Investigate if the sequence is arithmetic or geometric.

5. The n^{th}term of a sequence is given by $T_n=4n-2$. Verify that the sequence is arithmetic. Determine whether the sequence is increasing or decreasing.

6. Evaluate $\sum_{r=1}^{6}(3r+1)$.

7. Complex Numbers Revision

Let w = $\frac{(\cos\frac{\pi}{18}+i\sin\frac{\pi}{18})^{11}}{(\cos\frac{\pi}{36}+i\sin\frac{\pi}{36})^{4}}$. Express w in the form a+bi.

Seq + Series H/wk (2)

① $S_n = 3n^2 - 4n$ — find T_2 and T_3

(Arithmetic)

1st need a formula for T_n

$$S_n - S_{n-1} = T_n$$] to memorise.

$$3n^2 - 4n - [3(n-1)^2 - 4(n-1)] = T_n$$

$$3n^2 - 4n - [3n^2 - 6n + 3 - 4n + 4] = T_n$$

$$3n^2 - 4n - 3n^2 + 6n - 3 + 4n - 4 = T_n$$

$$6n - 7 = T_n$$

Now $T_1 = 6(1) - 7 = -1$

$T_2 = 6(2) - 7 = 5$

$T_3 = 6(3) - 7 = 11$] Answer

Q2

Let the 3 numbers be: $\frac{x}{r}$, x , xr.

Product = 216

$\frac{x}{r} \cdot x \cdot xr = 216$

$x^3 = 216$

$\boxed{x = 6}$

Sum = 19

$\frac{x}{r} + x + xr = 19$

$\frac{6}{r} + 6 + 6r = 19$ (x by r)

$6 + 6r + 6r^2 = 19r$

$6r^2 - 13r + 6 = 0$

$(3r - 2)(2r - 3)$

$r = 2/3$ $\quad$ $r = 3/2$.

$x = 6$ $\quad r = 3/2$

$\frac{6}{3/2}$, 6 , $6(3/2)$ = $\boxed{4, 6, 9}$

$x = 6$ $\quad r = 2/3$

$\frac{6}{2/3}$, 6 , $6(2/3)$ = $\boxed{9, 6, 4}$

Question 3

$\frac{1}{4}, a, 9$ Geometric

$\therefore \frac{a}{1/4} = \frac{9}{a}$

$a^2 = \frac{9}{4}$ $\therefore$ $\boxed{a = \frac{3}{2}}$

$\frac{1}{4}, a, 9-b$ Arithmetic

$\therefore a - \frac{1}{4} = 9 - b - a$ but $a = 3/2$

$\frac{3}{2} - \frac{1}{4} = 9 - b - \frac{3}{2}$

calc ...

$\boxed{b = \frac{25}{4}}$

4. $4x-4,\ x+4,\ 2x-7$

Check if Geom:

then $\frac{x+4}{4x-4} = \frac{2x-7}{x+4}$

$\therefore x^2+8x+16 = 8x^2-36x+28$

$0 = 7x^2-44x+12$

$(7x-2)(x-6)$

$x = 2/7$ $\quad x=6$ but $x \in \mathbb{Z}$ (integer/whole)

fill into 1st line: 20, 10, 5 yes geometric, $r = 1/2$.

If arithmetic:

then $x+4-4x+4 = 2x-7-x-4$

$-3x+8 = x-11$

$19 = 4x$

$19/4 = x$ but this isn't whole.

So not arithmetic.

(5) $T_n = 4n-2$

(i) Arithmetic? $T_n - T_{n-1}$ = a constant

$= 4n-2-[4(n-1)-2] =$

$= 4n-2-[4n-4-2] =$

$= 4n-2-4n+4+2 =$

$=$ 4 a constant Proven

(ii) Increasing or decreasing?

Since $d=4$ (Adding 4), the sequence is increasing.

(6) Evaluate $\sum_{r=1}^{6} 3r+1$ ie. find S_6.

$T_r = 3r+1$

$T_1 = 3(1)+1 = 4 = 9$

$T_2 = 3(2)+1 = 7$ $\Big\}\ d=3$ arithmetic

$T_3 = 3(3)+1 = 10$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_6 = \frac{6}{2}\left[2(4)+(6-1)3\right]$$

$= 3\left[8+15\right]$

$= 3\left[23\right] = 69$

(7) Complex Numbers:

$$\frac{\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^{11}}{\left(\cos\frac{\pi}{36} + i\sin\frac{\pi}{36}\right)^{4}}$$

$\frac{\pi}{18} = \frac{180}{18} = 10^\circ$

$\frac{\pi}{36} = \frac{180}{36} = 5^\circ$

$$\frac{\left(\cos 10^\circ + i\sin 10^\circ\right)^{11}}{\left(\cos 5^\circ + i\sin 5^\circ\right)^{4}}$$

1st Apply the powers

$$\frac{\cos 110^\circ + i\sin 110^\circ}{\cos 20^\circ + i\sin 20^\circ}$$

2nd Substract bottom angle from top angle

$\cos 90^\circ + i\sin 90^\circ$

$0 + 1i$

$= i$ = Answer