CLASS 5

Expected Value/Gaming

1

A game consists of rolling a fair six sided die. If the outcome of the game is 1 you win €2, if the outcome is 2 you win €4, if the outcome is 3 you win €6 and so on. It costs **€8** to roll the die once. Would you play this game to win money?

2

A school has a very good basketball team. Liam is the best player. He scores **58%** of his shots, each of which is worth **2 points.**

When Liam is fouled he gets two free shots called foul shots. He only scores **51%** of these shots which are worth **1 point each.**

The coach is concerned that opposing teams seem to be fouling Liam whenever he touches the ball and he asks you to work out if this could pay off for them in the long-run in terms of reducing Liam's scores.

(i) Calculate the expected value of the number of points Liam scores on one regular shot at the basket.

(ii) Calculate the expected value of the number of points Liam scores when he shoots two foul shots. Assume that all foul shots are independent events.

(iii) Which is better for the opposing team, that he is or is not fouled repeatedly?

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

In a competition Mary has a probability of $\frac{1}{20}$ of winning, a probability of $\frac{1}{10}$ of finishing in second place, and a probability of $\frac{1}{4}$ of finishing in third place. If she wins the competition she gets €9000. If she comes second she gets €7000 and if she comes third she gets €3000. In all other cases she gets nothing. Each participant in the competition must pay €2000 to enter.

(a) Find the **expected value** of Mary's loss if she enters the competition.

(b) Each of the 3 prizes in the competition above is increased by the same amount (€x) but the entry fee is unchanged.
For example, if Mary wins the competition now, she would get €$(9000 + x)$.
Mary now expects to break even.
Find the value of x.

Question 6 **(25 marks)**

A local sports club is planning to run a weekly lotto. To win the Jackpot of €1000, contestants must match one letter chosen from the 26 letters in the alphabet and two numbers chosen, in the correct order, from the numbers 0 to 9. In this lotto, repetition of numbers is allowed (e.g. M, 3, 3 is an outcome).

(a) Calculate the probability that M, 3, 3 would be the winning outcome in a particular week.

(b) If a contestant matches the letter only, or the letter and one number (but not both numbers), they will win €50. Using the table below, or otherwise, find how much the club should expect to make or lose on each play, correct to the nearest cent, if they charge €2 per play.

Event	**Payout (x) €**	**Probability ($P(x)$)**	$x.P(x)$
Win Jackpot			
Match letter and first number only			
Match letter and second number only			
Match letter and neither number			
Fail to win			

(c) The club estimates that the average number of plays per week will be 845. If the club wants to make an average profit of €600 per week from the lotto, how much should the club charge per play, correct to the nearest cent?

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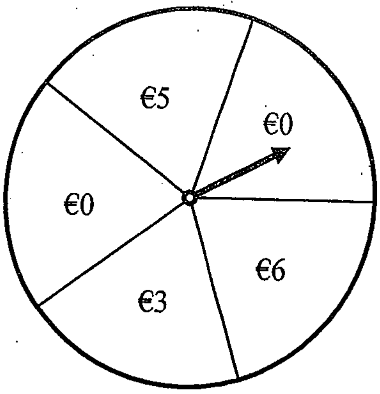
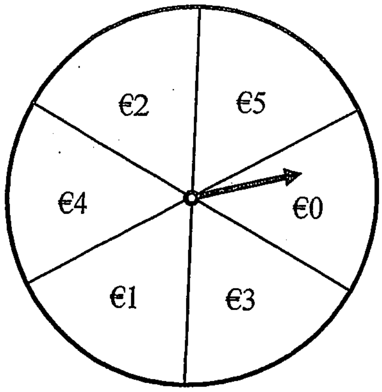
Question 3

(25 marks)

Two different games of chance, shown below, can be played at a charity fundraiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



Game A



Game B

(a) John played Game A four times and tells us that he has won a total of €8. In how many different ways could he have done this?

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for the charity? Give a reason for your answer.

(c) Mary plays Game B six times. Find the probability that the arrow stops in the €4 sector exactly twice.

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(a) 5, 3, 0, 0   3, 5, 0, 0   0, 5, 3, 0   0, 3, 5, 0

5, 0, 3, 0   3, 0, 5, 0   0, 5, 0, 3   0, 3, 0, 5

5, 0, 0, 3   3, 0, 0, 5   0, 0, 5, 3   0, 0, 3, 5

12 different ways

$\boxed{\text{OR}}$

Arranging 4 numbers $\longrightarrow$ $\dfrac{4!}{2!} = \dfrac{24}{2} = 12$ different ways.

$\div 2!$ since 2 zeros are the same $\longrightarrow$

if you swap the 2 zeros it doesn't look any different

(b) GAME A

Outcome/Payout	€0	€3	€5	€6
Probability	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
Expected Payout	$0(2/5)$	$3(1/5)$	$5(1/5)$	$6(1/5)$

Total Expected Payout = €$2\frac{4}{5}$ = €2.80

GAME B

Outcome/Payout	€0	€1	€2	€3	€4	€5
Probability	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$
Expected Payout	$0(1/6)$	$1(1/6)$	$2(1/6)$	$3(1/6)$	$4(1/6)$	$5(1/6)$

Total Expected Payout = €$2\frac{1}{2}$ = €2.50

GAME B pays out less So this is better for the charity.

(c) Success = lands on 4 $\therefore p = 1/6$

failure = not lands on 4 $\therefore q = 5/6$

plays 6 times $\therefore n = 6$

lands on 4 twice $\therefore r = 2$
(2 successes)

BERNOULLI TRIAL:

$$\binom{6}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4 = 0.2$$

So there is a 20% chance that the arrow stops at €4 twice