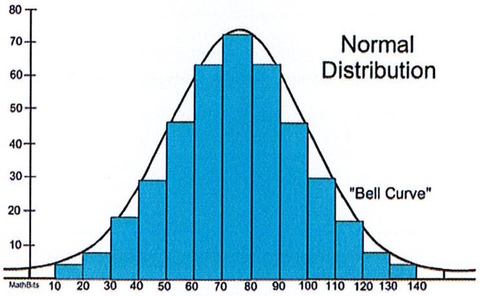
Class 2

Statistics/Probability

Z- Scores

Normal Distributions



Sample question about z-scores

A student got an exam in Physics and one in Chemistry. Here are the results-

SUBJECT	RESULT (%)	MEAN RESULT IN CLASS	STANDARD DEVIATION
Physics	60	55	1
Chemistry	70	65	2

In which subject did this student perform best?

Sample Question

The mean and standard deviation of 2 maths papers are shown. The mean marks for both papers are normally distributed.

Paper 1 Mean = 45 Standard Deviation = 8

Paper 2 Mean = 56 Standard Deviation = 12

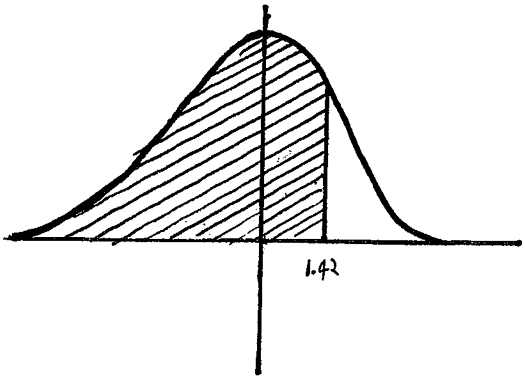
Sarah was present for Paper 1 but was absent for Paper 2. In Paper 1 she scored 59 marks.

(i) What was Sarahs z-score on Paper 1?

(ii) Sarah expected to do equally as well on Paper 2 . What would be the best estimate for her mark in this paper.

Probability and the Normal Distribution

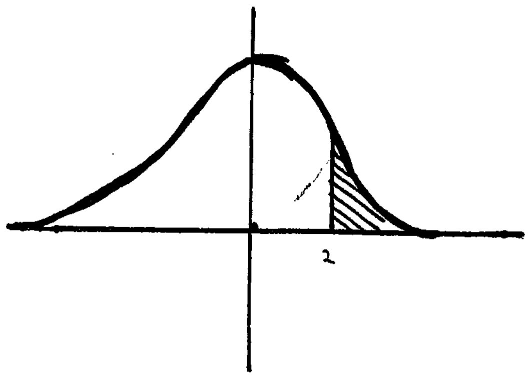
①



find $p(z \le 1.42)$

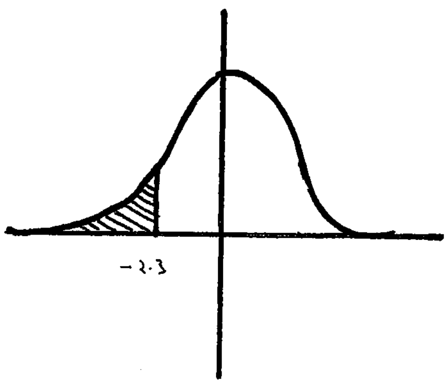
<u>ie.</u> the probability of getting a z-score less than or equal to 1.42

②



find $p(z \ge 2)$

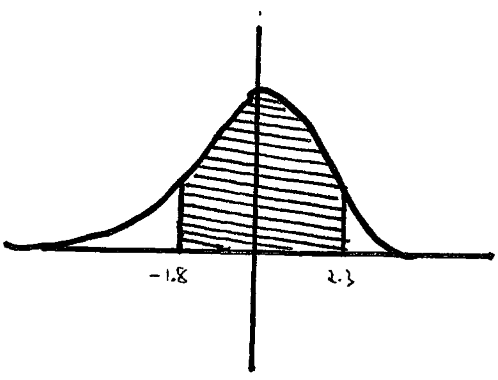
③



find $p(z \le -2.3)$

④

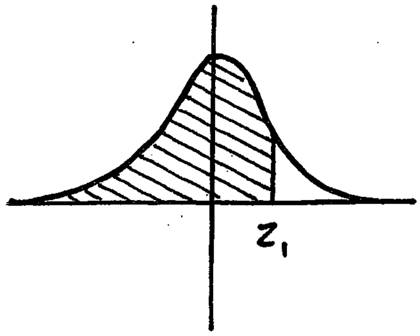
find $p(-1.8 \le z \le 2.3)$



Find the value of the z-score, z_1, in the following —

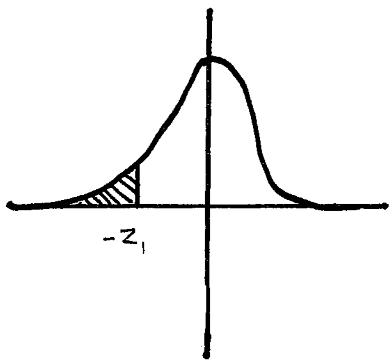
[1]

$$p(z \leq z_1) = .8686$$

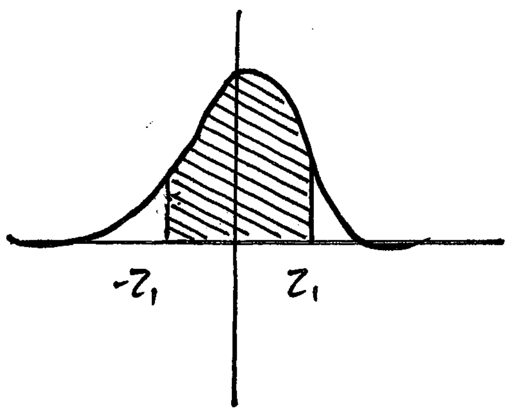


[2] $p(z \leq z_1) = .1724$

NOTE if the probability is less than .5, that means z_1 must be a negative z-score ie $-z_1$



[3] $p(-z_1 \leq z \leq z_1) = .6368$



Question 5 **(25 marks)**

A company produces calculator batteries. The diameter of the batteries is supposed to be 20 mm. The tolerance is 0·25 mm. Any batteries outside this tolerance are rejected. You may assume that this is the only reason for rejecting the batteries.

(a) The company has a machine that produces batteries with diameters that are normally distributed with mean 20 mm and standard deviation 0·1 mm. Out of every 10 000 batteries produced by this machine, how many, on average, are rejected?

(b) A setting on the machine slips, so that the mean diameter of the batteries increases to 20·05 mm, while the standard deviation remains unchanged. Find the percentage increase in the rejection rate for batteries from this machine.

In a large school, the heights of all third year students were recorded. The heights of the girls are normally distributed with a mean of 160cm and a standard deviation of 10cm. The heights of the boys are normally distributed with a mean of 164cm and a standard deviation of 12cm

(i) Find the probability that a girl chosen at random is taller than 168cm.

(ii) Nine per cent of the girls are shorter than x cm. Find the value of x.

(iii) Ninety per cent of the boys have heights between p cm and q cm, where p and q are symmetrical about 164 cm. Find the value of p and the value of q, where $p<q$.

[Pre. Paper 2015]

Statistics/Probability Homework

1 The mean height of all the students in a certain school is 175cm and the standard deviation is 15cm. If a student is selected at random, find the probability that he or she is less than or equal to 190cm tall.
If 90% of the students are less than or equal to Xcm, find the value of X assuming the distribution is normal.

2 The time taken by a postman to deliver letters to a certain apartment block is normally distributed with a mean of 12 minutes and standard deviation 2 minutes. Estimate the probability that he takes

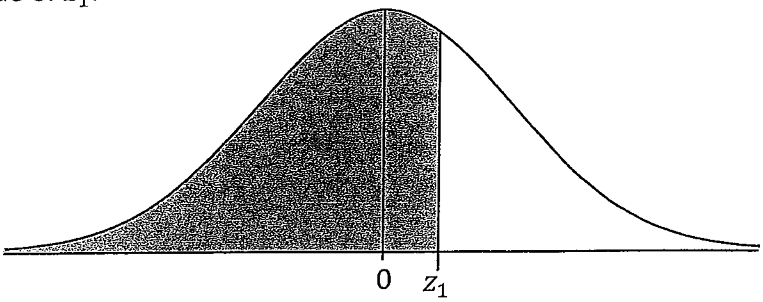
(i) Longer than 17 minutes
(ii) Less than 10 minutes
(iii) Between 9 and 13 minutes.

3 Find Z_1 if $P(-Z_1 \leq Z \leq Z_1) = 0.8438$

OVER

(25 marks)

(a) The diagram shows the standard normal curve. The shaded area represents 67% of the data. Find the value of z_1.



(b) The percentage results in a Maths exam for a class had a mean mark of 70 with a standard deviation of 15. The percentage results in an English exam for the same class had a mean mark of 72 with a standard deviation of 10. The results in both exams were normally distributed.

(i) Mary got 65 in Maths and 68 in English. In which exam did Mary do better relative to the other students in the class? Justify your answer.

Answer:

Justification:

(ii) In English the top 15% of students were awarded an A grade.
Find the least whole number mark that merited the award of an A grade in English.

(use z-scores)

(iii) Using the empirical rule, or otherwise, estimate the percentage of students in the class who scored between 52 and 82 in the English test.

Trig

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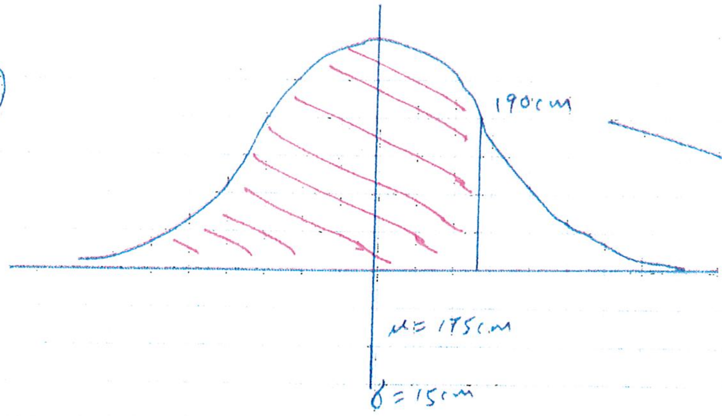
(1)



convert to a z-score

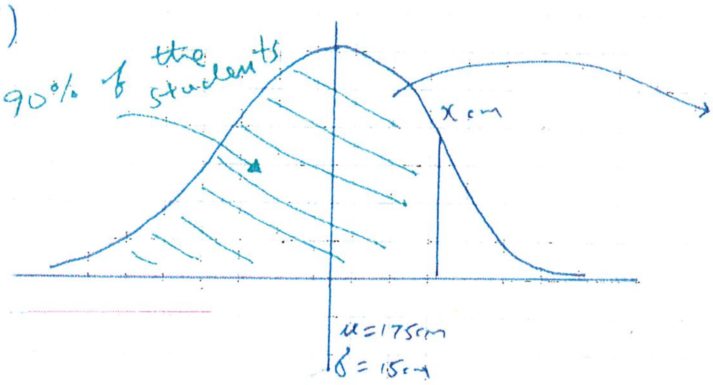
$$z = \frac{190 - 175}{15}$$

$$z = 1$$

(i) find probability $\leq$ 190cm

$\therefore P(z \leq 1) = \underline{\underline{.8413}}$ ANS.

(ii)



Probability of being in this section is .9

this approx corresponds to a z-score of 1.28

now convert z = 1.28 into its proper height x cm

$$z = \frac{x - \mu}{\sigma}$$

$$1.28 = \frac{x - 175}{15}$$

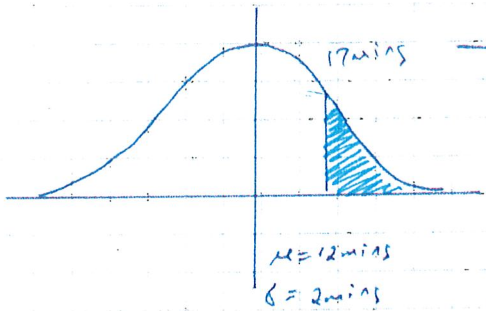
$$19.2 = x - 175$$

ANS: 194.2cm = x cm

Postman

(2)

(i) Longer than 17 minutes



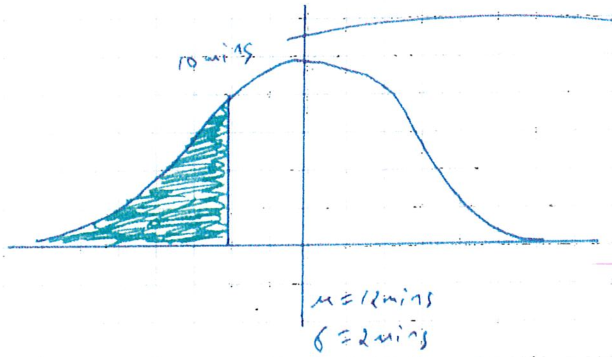
$Z = \frac{17-12}{2} = 2.5$

$P(Z > 2.5)$

$= 1 - (.9938) = \boxed{.0062}$

or $\boxed{.62\%}$

(ii) Less than 10 mins



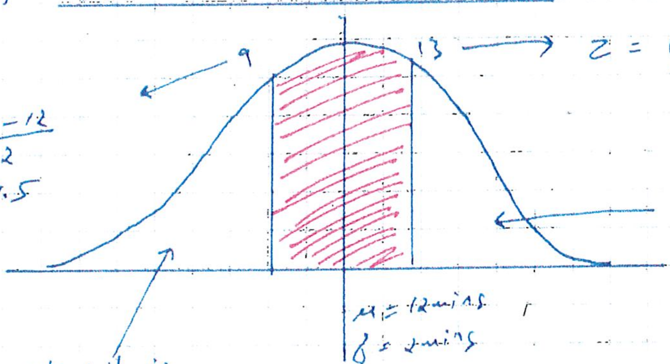
$Z = \frac{10-12}{2} = -1$

$P(Z \leq -1)$

$= 1 - (.8413)$

$\boxed{.1587}$ or $\boxed{15.87\%}$

(iii) Between 9 and 13 minutes



$Z = \frac{13-12}{2} = .5$

$Z = \frac{9-12}{2}$

$Z = -1.5$

white bit $= 1 - (.6915)$

$= \boxed{.3085}$

white bit is

$1 - (.9332)$

$\boxed{.0668}$

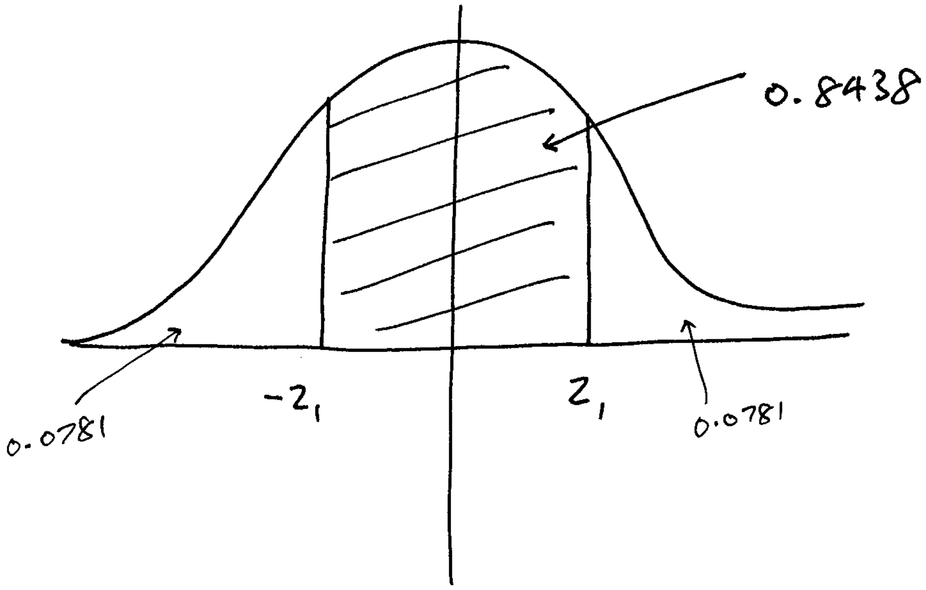
Red bit in between is $1 - (.0668 + .3085)$

Ans: $\boxed{.6247}$

or

$\boxed{62.47\% \text{ chance}}$

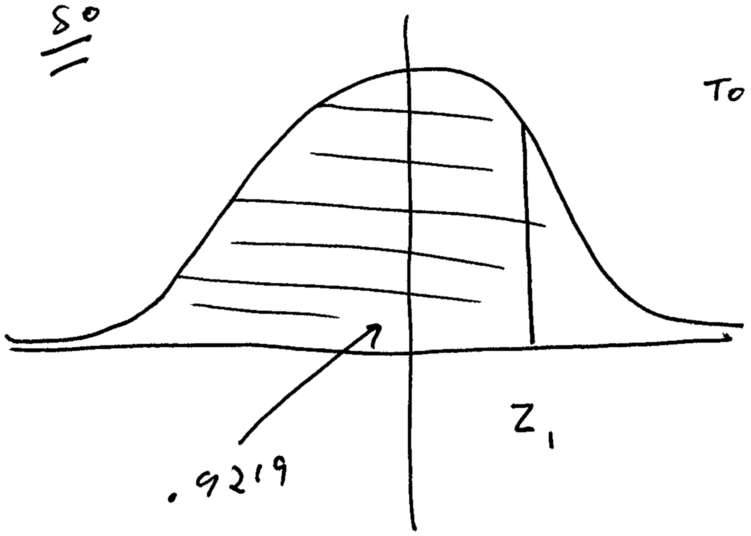
③ find Z_1 if $P(-Z_1 \le Z \le Z_1) = 0.8438$



$\overset{\text{Total}}{1} - \overset{\text{shaded}}{0.8438} = 0.1562 = 2$ Tails.

$0.1562 \div 2 = 0.0781 = 1$ Tail.

so



Total area to left of Z_1 is $.8438 + .0781$

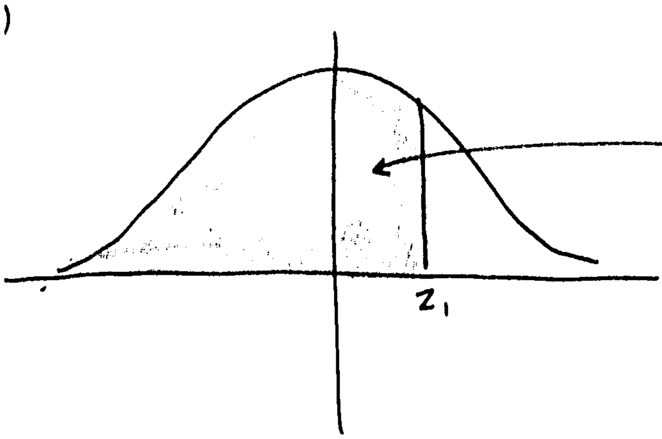
$= .9219$.

now look this up in logs and it corresponds to a Z-score of 1.42

Ans: $Z_1 = 1.42$.

L.C. 2018

(a)



67% = .6700

Go to logs ....
this corresponds to a z-score of 0.44

Ans: $z_1 = 0.44$

(b) Maths $\mu = 70$, $\sigma = 15$ English $\sigma = 10$, $\mu = 72$

(i)

$$z = \frac{65 - 70}{15}$$

$$z = -0.33$$

$$z = \frac{68 - 72}{10}$$

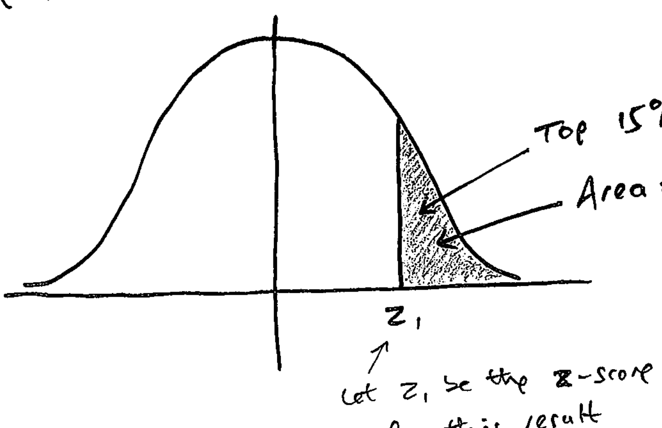
$$z = -0.4.$$

lower z-score.

Answer: did better in Maths

Justification: she got a lower z-score in English

(ii) English



$1 - p(z_1) = .15$

$1 - .15 = p(z_1)$

$.85 = p(z_1)$

this corresponds to a z-score of 1.04

Now find the actual result

$$z = \frac{x - \mu}{\sigma}$$

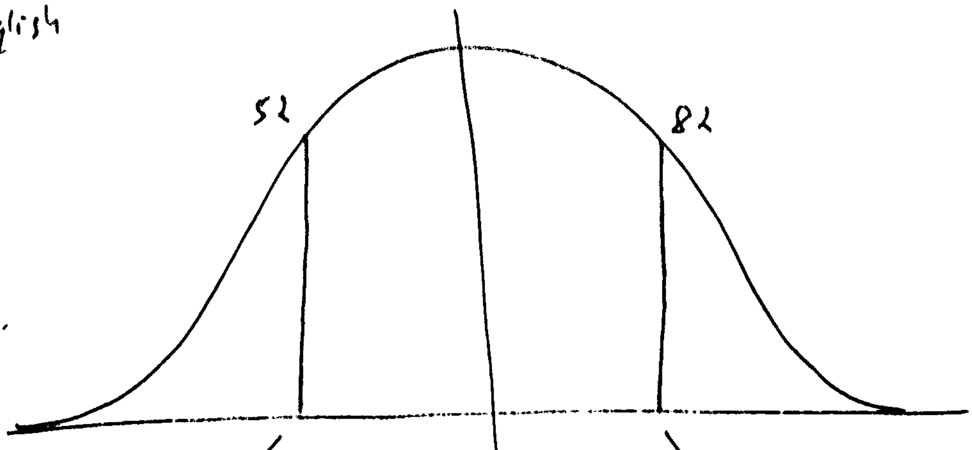
$$1.04 = \frac{x - 72}{10}$$

$$10.4 = x - 72$$

$$82.4 = x.$$

Ans: If you get over 82.4%, you get an A

(iii) English



$\mu = 72$

$\sigma = 10$

$z = \frac{52-72}{10}$

$z = -2$

$1 - (.9772)$

$= .0228$
(tail)

$z = \frac{82-72}{10}$

$z = 1$

$1 - (.8413)$

$= 0.1587$
(tail)

Shaded bit in between is

$1 - (.0228 + .1587)$

Ans: $= .8185$

$= 81.85\%$ got between 52 and 82.