Permutations and Combinations

1. (i) How many four-digit numbers can be made from the digits 1, 3, 4 and 5 (without repetition)?
   (ii) How many of these are odd?
   (iii) How many are even?
   (iv) How many are over 4,000?

2. How many ways are there of arranging the letters of the word DUBLINER:
   (i) If there are no restrictions
   (ii) If they must begin with B and end in L
   (iii) If the three vowels must be together
   (iv) If E and R must be together
   (v) If E and R must be apart

3. Four girls and five boys are to be lined up. How many arrangements are possible:
   (i) If there are no restrictions
   (ii) If two particular girls must be together
   (iii) If two particular boys must be kept apart
   (iv) If no two people of the same gender may be together

4. A class consists of six girls and seven boys. A team of four tennis players must be chosen from this class. How many different teams could be picked:
   (i) If there are no restrictions
   (ii) If the team must have two girls and two boys
   (iii) If the team must be all female

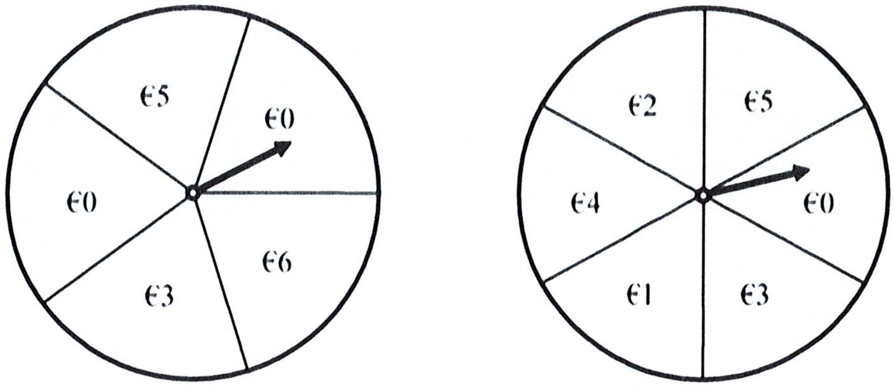
5. A committee of five persons is to be chosen from four women and seven men.
   If there must be more women than men on the committee, how many different committees could be formed?

6. In how many ways can a committee of five be chosen from 11 people:
   (i) If one particular person must be included
   (ii) If one particular person must be excluded
   (iii) If two particular people will not work on the same committee (that is, if one is included, the other must be excluded)

7. Twelve points are drawn on the circumference of a circle
   (i) How many different chords could be drawn between these points?
   (ii) How many triangles may be drawn by connecting these points?

Question 3 **(25 marks)**

Two different games of chance, shown below, can be played at a charity fundraiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



Game *A* Game *B*

(a) John played Game *A* four times and tells us that he has won a total of €8. In how many different ways could he have done this?

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for the charity? Give a reason for your answer.

(c) Mary plays Game *B* six times. Find the probability that the arrow stops in the €4 sector exactly twice.

page	running

Permutations and Combinations homework

❶ A code consists of a letter followed by 3 digits e.g. T044

(i) How many codes are possible?

(ii) If the 2^{nd} digit must be 2, how many codes are possible?

(iii) If the last digit is odd, how many different codes are possible?

❷ (i) How many different arrangements are possible from the letters of the word UKRAINE if there are no restrictions?

(ii) In how many of these arrangements do all 4 vowels appear side by side?

❸ (i) In how many different ways can 4 different cards be chosen from a standard pack of 52 cards?

(ii) How many of these choices contain the King of Diamonds?

❹ How many arrangements of the letters of the word SWARMED are possible if the A and R cannot be together?

① (i) | 26 | 10 | 10 | 10 |

26 letters

10 choices of number each time

Ans: $\boxed{26{,}000}$

• 10 digits available from 0-9.. $\underbrace{0,1,2,3,4,5,6,7,8,9}_{10 \text{ digits}}$

• repetition allowed

Security code eg.

| T | 0 | 4 | 4 |

can repeat digits

(ii) | 26 | 1 | 10 | 10 | = $\boxed{2{,}600}$ Ans

must be 2 ∴ 1 choice

(iii) | 26 | 10 | 10 | 5 | ← 5 choices = $\boxed{13{,}000}$ Ans.

last digit odd

ie, 1,3,5,7,9

② UKRAINE (7 letters)

(i) $7! = \boxed{5{,}040}$

(ii) 4 vowels together

$\underbrace{\text{UAIE}}_{1 \text{ thing}}$ K R N = 4 "things" = $4! \times 4!$

because you can re-arrange the 4 vowels each time also

$= \boxed{576}$

③ (i) $^{52}C_4 = \boxed{270{,}725 \text{ ways}}$

Pick 4 different cards

(ii) Must contain King of Diamonds ie its pre-selected

[only 51 cards available] → $^{51}C_3$ [only need to pick 3 other cards, King of Diamonds has already been picked]

$= \boxed{20{,}825 \text{ ways}}$

(4) SWARMED (7 letters)

A and R cannot be together

Rule: Total − [A and R together] = not together

$7! - 1{,}440$

$5{,}040 - 1{,}440$

$= \boxed{3{,}600 \text{ ways}}$

together

$\underbrace{AR}_{1\text{ thing}}$ S W M E D = 6 things

$= 6! \times 2! \leftarrow$ (swap A and R)

$= 1{,}440$ ways