CLASS 2

Integration Rates of Change

Sample paper

Raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an initial downward velocity of $10ms^{-1}$ and its downward acceleration is given by :

$$a(t) = \begin{cases} 9-0.9t & if\ 0 \le t \le 10 \\ 0 & for\ t > 10 \end{cases}$$

The raindrop is initially 500m above the ground.

(i) Find the velocity function V(t) of the raindrop after t seconds, $0 \le t \le 10$.

(ii) Find V(10) and hence, write down the speed at which the raindrop hits the ground.

(iii) Find S(t), the distance travelled by the raindrop in t seconds, $0 \le t \le 10$.

(iv) How long does it take the raindrop to fall?

Integration Homework

Question 9 **(50 marks)**

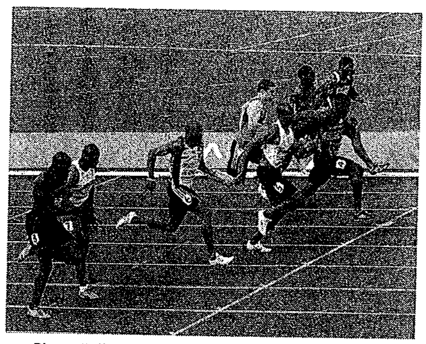
(a) Let $f(x) = -0{\cdot}5x^2 + 5x - 0{\cdot}98$, where $x \in \mathbb{R}$.

(i) Find the value of $f(0{\cdot}2)$.

(ii) Show that f has a local maximum point at $(5,\ 11{\cdot}52)$.

(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

$$v(t) = \begin{cases} 0, & \text{for } 0 \le t < 0{\cdot}2 \\ -0{\cdot}5t^2 + 5t - 0{\cdot}98, & \text{for } 0{\cdot}2 \le t < 5 \\ 11{\cdot}52, & \text{for } t \ge 5 \end{cases}$$



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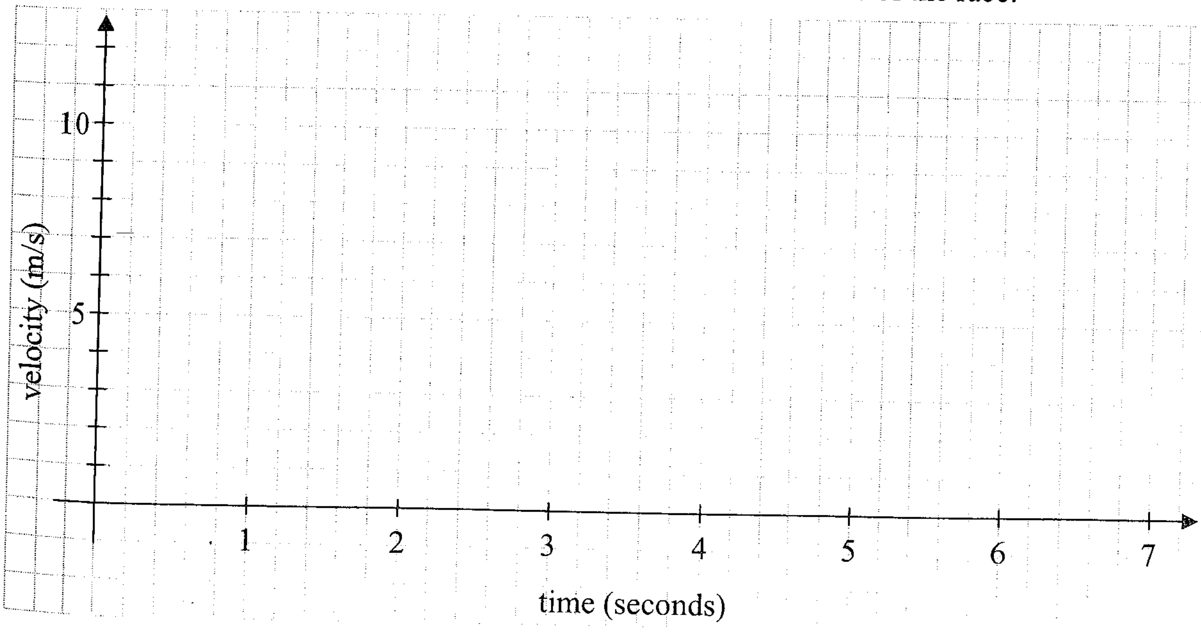
Note that the function in part **(a)** is relevant to $v(t)$ above.

(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.

(iii) Find the sprinter's finishing time for the race. Give your answer correct to two decimal places.

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2014 SEC SAMPLE
PAPER 1

Question 9 **(50 marks)**

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$f(t)=12\cdot25+4\cdot75\sin\left(\frac{2\pi}{365}t\right),$$

where t is the number of days after March 21st and $\left(\frac{2\pi}{365}t\right)$ is expressed in radians.

(a) Find the length of the day in Galway on June 5th (76 days after March 21st). Give your answer in hours and minutes, correct to the nearest minute.

(b) Find a date on which the length of the day in Galway is approximately 15 hours.

(c) Find $f'(t)$, the derivative of $f(t)$.

(d) Hence, or otherwise, find the length of the longest day in Galway.

(e) Use integration to find the average length of the day in Galway over the six months from March 21st to September 21st (184 days). Give your answer in hours and minutes, correct to the nearest minute.

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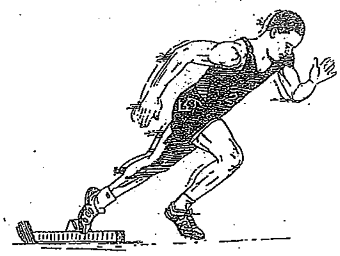
Question 8 **(55 marks)**

(a) In the 100-metre race, sprinters typically reach their top speed about halfway through the race and try to maintain that speed for as long as possible.



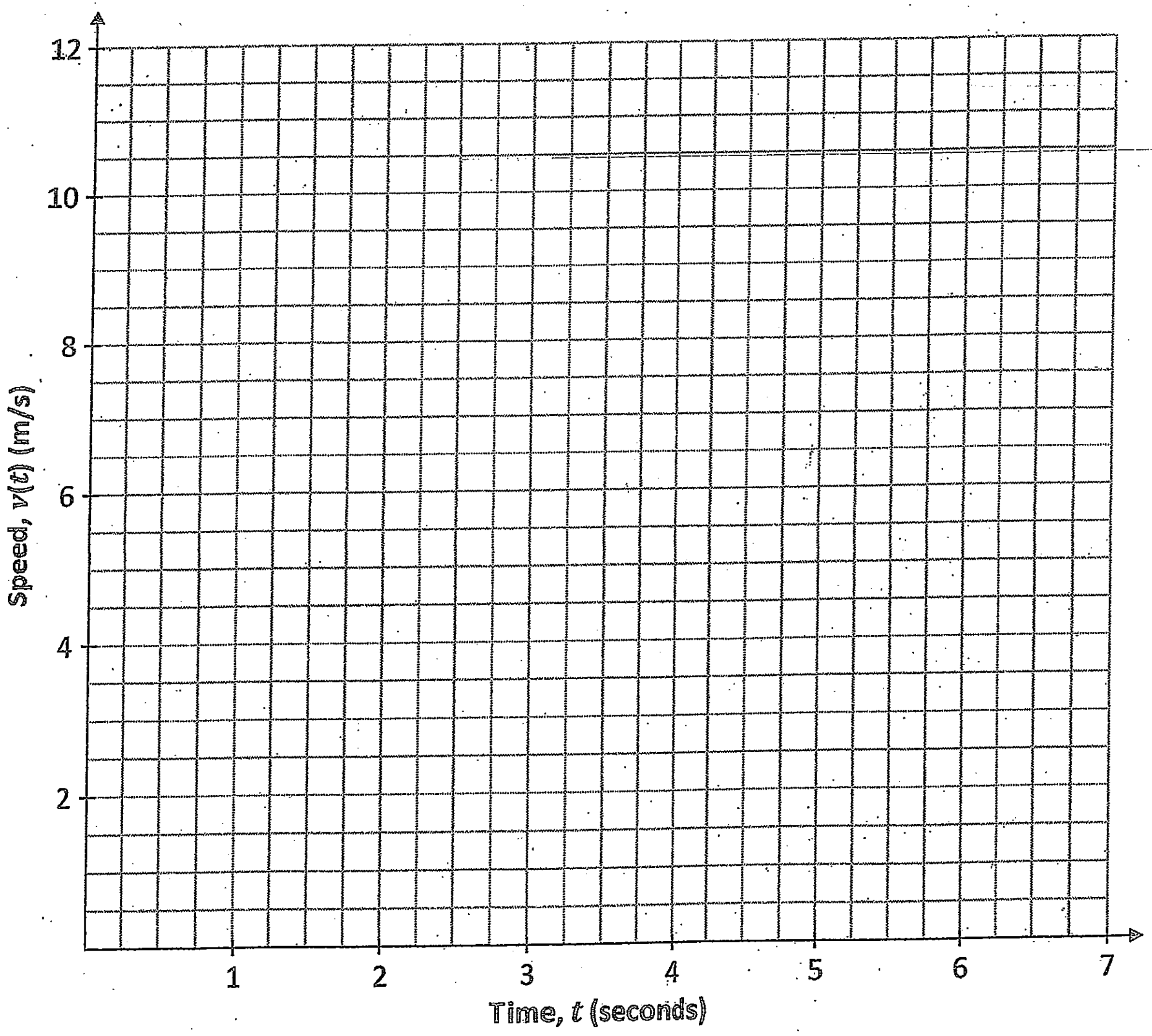
A student analysed a sprinter's performance over the course of a particular race and determined that the speed of the sprinter can be approximated by the following model:

$$v(t) = \begin{cases} 0, & 0 \le t < 0{\cdot}15 \\ -0{\cdot}6t^2 + 5{\cdot}4t - k, & 0{\cdot}15 \le t < 4{\cdot}5 \\ 11{\cdot}3535, & t \ge 4{\cdot}5 \end{cases}$$

where v is the speed in metres per second, t is the time in seconds from the starting signal and k is a constant.

(i) Find the value of k.

(ii) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(iii) Find the distance travelled by the sprinter in the first 4·5 seconds of the race.

(iv) Hence, find the sprinter's finishing time for the race. Give your answer correct to three decimal places.

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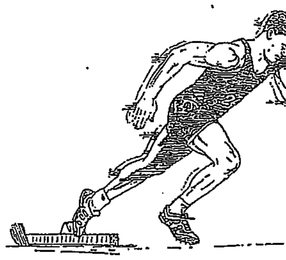
Question 8 **(55 marks)**

(a) In the 100-metre race, sprinters typically reach their top speed about halfway through the race and try to maintain that speed for as long as possible.



A student analysed a sprinter's performance over the course of a particular race and determined that the speed of the sprinter can be approximated by the following model:

$$\text{(Speed)}\quad v(t) = \begin{cases} 0, & 0 \le t < 0{\cdot}15 \\ -0{\cdot}6t^2 + 5{\cdot}4t - k, & 0{\cdot}15 \le t < 4{\cdot}5 \\ 11{\cdot}3535, & t \ge 4{\cdot}5 \end{cases}$$

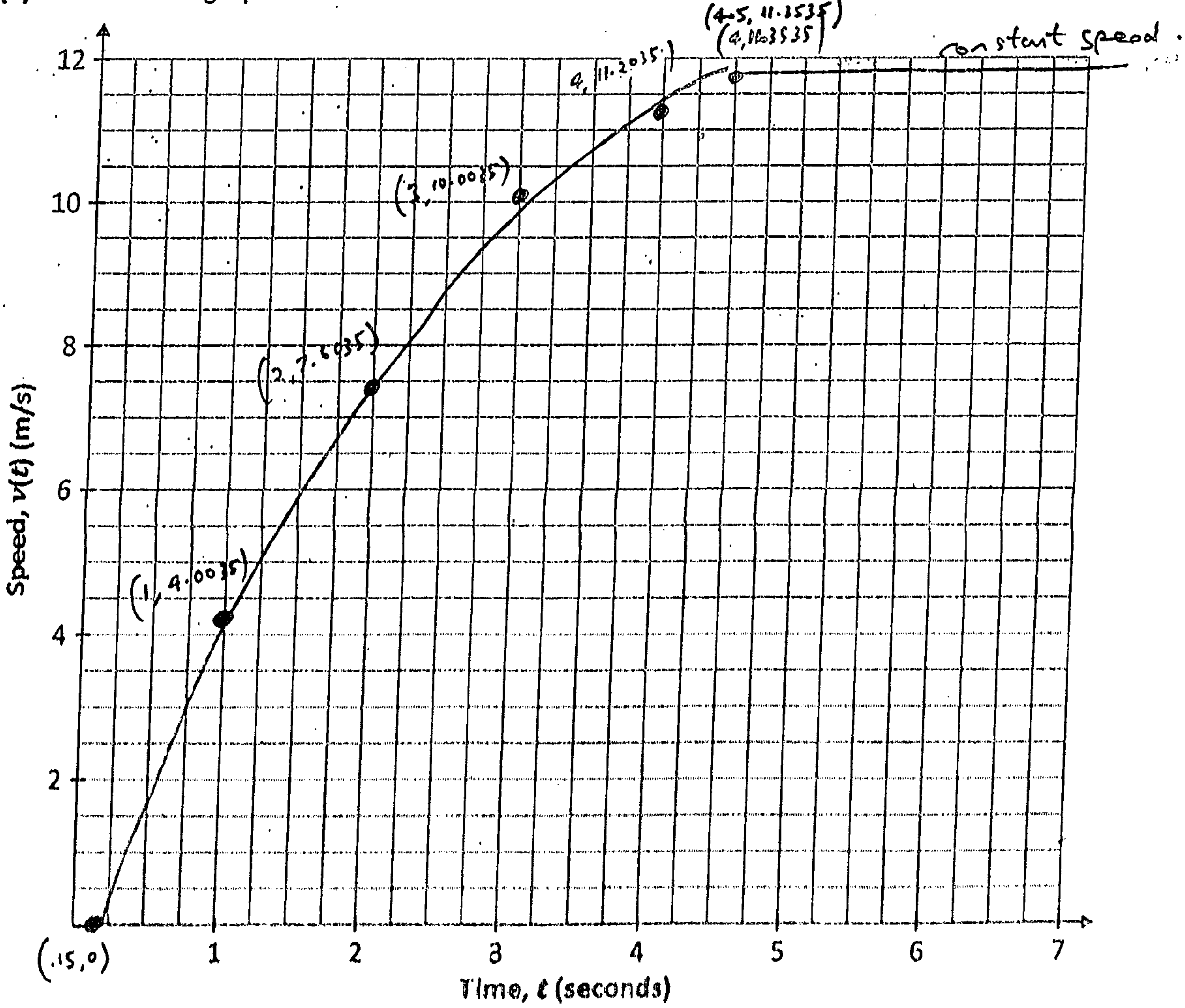
where v is the speed in metres per second, t is the time in seconds from the starting signal and k is a constant.

(i) Find the value of k. when $t = 4.5$ $v(t) = 11.3535$ from above.

$11.3535 = -0.6(4.5)^2 + 5.4(4.5) - k$

calc ... $\boxed{k = 0.7965}$

(ii) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(iii) Find the distance travelled by the sprinter in the first 4·5 seconds of the race.

Distance $= \int$ speed

$= \int (-0.6t^2 + 5.4t - 0.7965)\,dt$

$= -\frac{0.6t^3}{3} + \frac{5.4t^2}{2} - 0.7965t \Big]_{.15}^{4.5}$

$= -.2t^3 + 2.7t^2 - 0.7965t \Big]_{.15}^{4.5}$

$= -.2(4.5)^3 + 2.7(4.5)^2 - 0.7965(4.5) - [-.2(.15)^3 + 2.7(.15)^2 - 0.7965(.15)]$

Ans: $\underline{\underline{32.93}}$ metres.

(iv) Hence, find the sprinter's finishing time for the race. Give your answer correct to three decimal places.

$100 - 32.93 = 67.07$ metres left.

takes 4.5 secs.

speed is constant 11.3535 for this part of race

Time $= \frac{\text{Dist}}{\text{Speed}} = \frac{67.07}{11.3535} \simeq 5.9$ secs

Total time $= 5.9 + 4.5$

Ans 10.4 seconds

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