CLASS 4

Question 8 **(55 marks)**

(a) When a loan of €P is repaid in equal repayments of amount €A, at the end of each of t equal periods of time, where i is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$A = P\frac{i(1+i)^t}{((1+i)^t - 1)}$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

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Question 9 **(50 marks)**

Mary and John took out a mortgage of €400 000 to buy a new house. They agreed to repay this loan, plus interest, by a series of equal monthly payments, starting one month after they received the loan and continuing for 20 years. The effective annual rate of interest charged is 6%.

(a) **(i)** Show that the rate of interest, compounded monthly, that corresponds to an effective annual rate of 6% is 0·487%, correct to three decimal places.

(ii) Show how to use the sum of a geometric series to calculate the monthly repayments that Mary and John have to make on their mortgage, correct to the nearest cent.

(iii) Using amortisation, or otherwise, verify your answer to part **(ii)** above.

(b) After two years of paying their monthly repayments on their mortgage in full, Mary and John's financial situation worsened and they were unable to make any further repayments.

(i) Using amortisation, or otherwise, find the amount outstanding immediately after Mary and John made their last repayment. Give your answer correct to the nearest cent.

(ii) Mary and John's lender charges an additional rate of interest of 1·5% per annum on the outstanding amounts of loans in arrears, compounded monthly.
How long will it take for the outstanding amount that Mary and John owe to exceed the original sum of money borrowed?

(iii) Give one justification why Mary and John's lender would charge a higher rate of interest on loans in arrears.

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Question 6 **(25 marks)**

Tara is planning for her retirement and wishes to calculate the value of the pension fund she requires when she retires. She wants to have a pension fund that could, from one month after the date of her retirement, give her a monthly payment of €2000 for 25 years, assuming the fund will earn an annual equivalent rate (AER) of 3·5% over the period of the payments.

(a) **(i)** Show that the rate of interest, compounded monthly, which is equivalent to an AER of 3·5% is 0·287%, correct to three decimal places.

(ii) Find, correct to the nearest euro, the value of the pension fund that Tara requires when she retires.

(4)

(b) Tara decides that she wants the pension fund to give her a fixed monthly payment indefinitely. Assuming the fund will continue to earn an AER of 3·5%, find the amount of each payment, correct to the nearest cent.

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H/W 4

Question 4 **(25 marks)**

Dan and Kate plan to buy a house which costs €250 000. In order to get a mortgage on the property, the couple need to save a deposit of 10% of the purchase price. They open a savings account in their local Credit Union which offers an annual equivalent rate (AER) of 3·5%.

(a) **(i)** Show that the rate of interest, compounded monthly, which is equivalent to an AER of 3·5% is 0·287%, correct to three decimal places.

(ii) Dan and Kate decide to put €500 in the savings account at the beginning of each month. How long will it take them to save up the deposit for the house? Give your answer in months, correct to the nearest month.

(b) After saving for three years, Dan and Kate find the perfect house. They decide to borrow the remainder of the **deposit** at a monthly interest rate of 0·425%, fixed for the term of the loan. The loan is to be repaid in equal monthly repayments over five years and the first repayment is due one month after the loan is issued. Calculate the amount of each monthly repayment, correct to the nearest cent.

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Financial Maths Pre. 2018 Dan + Kate

Q4

(a) (i) $r = (1.035)^{\frac{1}{12}} - 1 = 0.00287 = 0.287\%$ Proven

(ii)

let n = the number of months they need to save for

Deposit = 10% of €250,000 = €25,000.

Start month 1	Start Month 2	....	Start month n		
save €500	Save €500		save €500	=	€25,000

In the future,
the sum of all these €500 lodgements needs to equal €25,000

Apply $F = P(1+i)^t$ to each one .....

$$500(1.00287)^n + 500(1.00287)^{n-1} + \dots + 500(1.00287)^1 = 25,000$$

the first €500 will be saved for the full n months

this has 1 month less so $n-1$

this goes in at start of last month, so only has 1 month to be saved.

$$500\left[(1.00287)^1 + \dots \dots + (1.00287)^n\right] = 25,000$$

$$500\left[\begin{array}{l} a = 1.00287 \\ r = 1.00287 \end{array} \quad S_n = \frac{a(1-r^n)}{1-r}\right] = 25,000$$

$$500\left[\frac{1.00287\left(1-(1.00287)^n\right)}{1-1.00287}\right] = 25,000$$

$\longrightarrow$

$$500\left[-349.4320557\left(1-(1.00287)^n\right)\right] = 25{,}000$$

$$\div$$

$$-349.4320557\left(1-(1.00287)^n\right) = 50$$

$$\div$$

$$1-(1.00287)^n = -0.143089333$$

$$1.143089333 = (1.00287)^n$$

$$\ln(1.143089333) = \ln(1.00287)^n$$

$$\ln(1.143089333) = n\ln(1.00287)$$

$$46.66 = n$$

$$47 \text{ months} = n$$

(b) saved £500 each month for 3 years = 36 months.

So first find the sum of the 1st 36 terms of the geometric series from a (ii)

$$S_{36} = 500\left[\frac{1.00287\left(1-(1.00287)^{36}\right)}{1.00287}\right]$$

$$\underline{\underline{S_{36} = €18,988.51}}$$] they save have managed to this after 36 months.

<u>Remainder of Deposit</u> = €25,000 − €18,988.51

= $\underline{\underline{€6,011-49}}$ (they need to borrow this)

5 year loan = 60 months

APR = 0.00425

<u>Find the monthly repayments</u>

Amortisation − $$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$A = 6,011.49\frac{(0.00425)(1.00425)^{60}}{(1.00425)^{60} - 1}$$

$$A = \underline{€113.72 \text{ per month}}$$

Question 9 **(50 marks)**

Mary and John took out a mortgage of €400 000 to buy a new house. They agreed to repay this loan, plus interest, by a series of equal monthly payments, starting one month after they received the loan and continuing for 20 years. The effective annual rate of interest charged is 6%.

(a) **(i)** Show that the rate of interest, compounded monthly, that corresponds to an effective annual rate of 6% is 0·487%, correct to three decimal places.

$$r = (1.06)^{\frac{1}{12}} - 1$$

$$r = .00487 = 0.487\%$$

PROVEN.

(ii) Show how to use the sum of a geometric series to calculate the monthly repayments that Mary and John have to make on their mortgage, correct to the nearest cent.

Let A = amount of each monthly repayment

20 yrs = 240 months

$$400{,}000 = \frac{A}{(1.00487)^1} + \frac{A}{(1.00487)^2} + \dots + \frac{A}{(1.00487)^{240}}$$

$$400{,}000 = A\left[S_{240} \right]$$

$$\frac{400{,}000}{S_{240}} = A$$

€2,829.84 = Amount of each repayment.

(iii) Using amortisation, or otherwise, verify your answer to part (ii) above.

$$A = \frac{400{,}000(.00487)(1.00487)^{240}}{(1.00487)^{240} - 1}$$

A = €2,829.84 also.

Question 6 **(25 marks)**

Tara is planning for her retirement and wishes to calculate the value of the pension fund she requires when she retires. She wants to have a pension fund that could, from one month after the date of her retirement, give her a monthly payment of €2000 for 25 years, assuming the fund will earn an annual equivalent rate (AER) of 3·5% over the period of the payments.

ie at end of each month.

(a) (i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of 3·5% is 0·287%, correct to three decimal places.

$r = (1.035)^{\frac{1}{12}} - 1$

$r = 0.0028708$

$r = 0.002871$

$= .287\%$

(ii) Find, correct to the nearest euro, the value of the pension fund that Tara requires when she retires.

25 years × 12 = 300 months

Rule ⇒ Pension Fund = Sum of present values of all the pension payments

Pension $= \frac{2{,}000}{(1.00287)^1} + \frac{2{,}000}{(1.00287)^2} + \dots + \frac{2{,}000}{(1.00287)^{300}}$

$= 2{,}000 \left[S_{300} \quad a = \frac{1}{1.00287} \quad r = \frac{1}{1.00287} \right]$

$= 2{,}000 \left[200.95 \right]$

= €401, 909 needed.